Heat Transfer from a Spherical Particle in a Rarefied Monatomic Gas

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The problem of heat transfer from a spherical particle situated in an infinite expanse of a monatomic gas is considered. The Bhatnagar, Gross, and Krook (BGK) model of the linearized Boltzmann equation is used. The boundary-value problem is converted to a system of linear integral equations, which is solved numerically. Results for radial heat flux, gaseous density, and temperature for all degrees of rarefaction, and arbitrary thermal accommodation coefficient, are obtained.

Introduction

THE problem of heat transfer from a sphere in a rarefied gas has been studied both experimentally and theoretically by a number of investigators. On monatomic gases, the available work includes the theoretical investigations of Takao,[1] Lees,[2] Springer and Tsai,[3] Cercignani and Pagani,[4] and the experimental measurements of thermal accommodation coefficients of helium, argon, and xenon gases on Zircaloy-2 and UO_2 spheres at room temperature by Thomas and Loyalka.[5,6]

In this paper, we extend the previous theoretical work and, in addition to the radial heat flux, we report results for density and temperature perturbations of the gas in the neighborhood of the sphere. The problem is solved using the BGK[7] model of the linearized Boltzmann equation. We convert the relevant integro-differential equation with associated boundary conditions into a system of integral equations that are then solved numerically.

The reported results for the heat flux are more accurate in the near free molecule and transition regimes than those reported previously. The results for the profiles are new and should encourage further experimental and theoretical studies of the problem.

Statement of the Problem

Consider a sphere of radius $\tilde{R}$ at rest in an infinite expanse of a monatomic gas. The surface of the sphere is maintained at a constant temperature T_s, different from the gas temperature T_∞, far away from the sphere. Let $\boldsymbol{r}$ and $\boldsymbol{c}$ be, respectively, the nondimensional position and velocity vector of a gaseous molecule. If $(T_s - T_\infty)/T_\infty$ is small, the Boltzmann equation for the distribution function $f(\boldsymbol{r}, \boldsymbol{c})$ can be linearized by writing $f = f_\infty(1+h)$. Here, h is a measure of the perturbation on the distribution function from the local Maxwellian f_∞, $\|h\| \ll 1$.

Assuming the BGK model, the steady-state form of the linearized Boltzmann equation in the absence of external forces leads to the following nondimensional boundary-value problem:[4]

$$\boldsymbol{c} \cdot \frac{\partial h}{\partial \boldsymbol{r}} = Lh \tag{1}$$

where

$$Lh = -h(\boldsymbol{r}, \boldsymbol{c}) + \sum_{m=1}^{2} \psi_m(\boldsymbol{c})\, a_m(\boldsymbol{r})$$

with the boundary conditions

$$h^+ = \gamma + \tau(c^2 - 2), \qquad \boldsymbol{r} \in \partial S,\ \boldsymbol{c} \cdot \boldsymbol{n}_r > 0 \tag{2}$$

$$h(\boldsymbol{r}, \boldsymbol{c}) \to 0 \qquad \text{for } |\boldsymbol{r}| \to \infty \tag{3}$$

where the functions ψ_m are given by

$$\psi_1 = 1 \tag{4}$$

$$\psi_2 = (2/3)[c^2 - (3/2)] \tag{5}$$

The moments $a_m(\boldsymbol{r})$ are expressed as

$$a_m(\boldsymbol{r}) = (\rho_m, h) = \int \mathrm{d}\boldsymbol{c} \frac{\exp(-c^2)}{\pi^{3/2}} h(\boldsymbol{r}, \boldsymbol{c}) \rho_m(\boldsymbol{c}) \tag{6}$$

and the functions ρ_m are defined as

$$\rho_1(\boldsymbol{c}) = 1 \tag{7}$$

$$\rho_2(\boldsymbol{c}) = c^2 - 3/2 \tag{8}$$

Further, the constants γ and τ are given by the expressions

$$\gamma = [(1, h^-)]_b \kappa \tag{9}$$

$$\tau = \{\alpha_t [(1 - \alpha_t)/2] [(c^2 - 2, h^-)]_b\} \kappa \tag{10}$$

where

$$\kappa = \frac{\Delta T}{T_\infty} \tag{11}$$

and $\Delta T = T_s - T_\infty$. We note that the linearization is valid only if $\kappa \ll 1$ but that, in the linearized problem, κ appears as a multiplicative constant in the boundary conditions. Thus, the perturbation h and the macroscopic quantities will be proportional to κ. Therefore, without any loss of generality, when convenient in the future, we shall take $\kappa = 1$.

We also note that α_t is the thermal accommodation coefficient, and the scalar product is given by

$$[(\rho, h^-)]_b = \frac{2}{\pi} \int \mathrm{d}\boldsymbol{c} \exp(-c^2) |c_r| \rho h^- \tag{12}$$

Here, $c_r = \boldsymbol{c} \cdot \boldsymbol{n}_r$ is the radial component of the velocity, and $\boldsymbol{n}_r$ is a unit normal in the radial direction. h^+ and h^- correspond, respectively, to the outward (from the sphere into the gas) and inward (from the gas to the sphere) molecular velocity distributions at the surface of the sphere.

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The nondimensional variables are defined as

$$r = \frac{\tilde{r}}{\ell_t}, \qquad R = \frac{\tilde{R}}{\ell_t} \tag{13}$$

$$c = \left(\frac{m}{2kT_\infty}\right)^{1/2} \tilde{c} \tag{14}$$

where the mean free path ℓ_t is given by

$$\ell_t = \frac{4}{5}\frac{\lambda}{p_\infty} T_\infty \left(\frac{m}{2\kappa T_\infty}\right)^{1/2} \tag{15}$$

and λ is the gas thermal conductivity; p_∞ and κ are, respectively, the pressure of the gas far away from the sphere and the Boltzmann constant.

The moments $a_1(r)$ and $a_2(r)$, Eq. (6), are, respectively, the density and temperature perturbations:

$$a_1(r) = \frac{n(r) - n_\infty}{n_\infty} \tag{16}$$

$$a_2(r) = \frac{T(r) - T_\infty}{T_\infty} \tag{17}$$

Also, the radial heat flux is expressed as

$$\tilde{q}(r) = p_\infty \left(\frac{2kT_\infty}{m}\right)^{1/2} q(r) \tag{18}$$

where the dimensionless radial heat flux $q(r)$ is given by

$$q(r) = \frac{1}{\pi^{3/2}} \int \mathrm{d}c\, c_r \left(c^2 - \frac{5}{2}\right) e^{-c^2} h(r, c) \tag{19}$$

Asymptotic Solutions

A. Continuum Limit

In the continuum limit ($Kn \ll 1$), the solution to Eq. (1) is given by the classical Chapman-Enskog theory in the form:[4]

$$h_{\mathrm{asy}}(r, c) = \frac{T(r) - T_\infty}{T_\infty}(c^2 - 5/2) + c \cdot \frac{\nabla T(r)}{T_\infty} A(c) \tag{20}$$

where $A(c)$ is the solution of the integral equation

$$L(A(c) c_r) = c_r(c^2 - 5/2) \tag{21}$$

and L is the BGK model operator. The function $A(c)$ is in fact given by

$$A(c) = -(c^2 - 5/2) \tag{22}$$

From Eqs. (6), (18), and (22), the heat flux in this limit becomes

$$\tilde{q}(r) = -\lambda \frac{\partial T}{\partial \tilde{r}} \tag{23}$$

B. Free Molecular Limit

In the free molecular limit ($Kn \gg 1$), the distribution function h is given by

$$h = \alpha_t (c^2 - 2)\kappa \qquad \text{for } \sin\theta \le r_0/r, \qquad 0 \text{ otherwise} \tag{24}$$

Equations (18) and (24) yield the total radial heat flux in this regime, as

$$\tilde{q}_{fm}(r) = \frac{R^2}{r^2} \cdot \alpha_t \cdot \frac{1}{2} p_\infty \left(\frac{8kT_\infty}{\pi m}\right)^{1/2} \kappa \tag{25}$$

Method of Solution for Arbitrary Knudsen Number

Integrating Eq. (1) along the characteristic path s, we obtain

$$h(r, c) = \int_0^\infty \frac{1}{c} \exp(-s''/c) \sum_{m=1}^{2} \psi_m(c)\, a_m(r')\, \mathrm{d}s'' + h(R, c) \exp(-s/c) \tag{26}$$

where $r' = r - s''\Omega$, $s'' = |r - r'|$, and $\Omega = c/c$. Here, the last term is nonzero only for Ω along directions joining R to r.

Taking moments of Eq. (26) with respect to $\rho_m(c)$, we obtain the integral equations

$$a_m(r) = \gamma S_m(r) + \tau J_m(r) + \sum_{j=1}^{2} K_{mj}(r, r')\, a_j(r')\, \mathrm{d}r', \qquad m = 1,2 \tag{27}$$

where the source terms S_m and J_m are given by

$$S_m(r) = [\rho_m(c), 1]_s \tag{28}$$

$$J_m(r) = [\rho_m(c), (c^2 - 2)]_s \tag{29}$$

Here the inner product is defined as

$$(\rho_m, f)_s = \frac{1}{\pi^{3/2}} \int \mathrm{d}c \exp(-c^2 - |r - R|/c)\, \rho_m f \tag{30}$$

Also, the kernel $K_{mj}(r, r')$ is given by

$$K_{mj}(r, r') = \frac{2r'}{\pi^{1/2} r} \int_{|r-r'|}^{(r^2-R^2)^{\frac{1}{2}} + (r'^2 - R^2)^{\frac{1}{2}}} \mathrm{d}t/t \int_0^\infty c\, \mathrm{d}c \exp(-c^2 - t/c)\, \rho_m(c)\, \psi_j(c) \tag{31}$$

The source terms and kernels are further simplified into forms suitable for computations (see Appendix A). The functions $a_m(r)$ can be written as

$$a_m(r) = \gamma P_m(r) + \tau Q_m(r) \tag{32}$$

where the functions $P_m(r)$ and $Q_m(r)$ are determined from the integral equations

$$P_m(r) = S_m(r) + \sum_{j=1}^{2} \int_{r_0}^{\infty} K_{mj}(r, r') P_j(r')\, \mathrm{d}r' \tag{33}$$

$$Q_m(r) = J_m(r) + \sum_{j=1}^{2} \int_{r_0}^{\infty} K_{mj}(r, r') Q_j(r')\, \mathrm{d}r' \tag{34}$$

Integral equations (33) and (34) were solved using an 81-point Gauss-Kronrod quadrature and the numerical method described in Refs. 8–11. Once $P_m(x)$ and $Q_m(x)$ are known, the complete solution of Eq. (27) can be constructed through determination of γ and τ by use of Eqs. (9) and (10) and (26–32). Using these equations, one finds

$$\begin{pmatrix} \gamma \\ \tau \end{pmatrix} = \underline{\underline{D}}^{-1} \begin{pmatrix} 0 \\ \alpha_t \end{pmatrix} \kappa \tag{35}$$

The elements of matrix D are given in Appendix B and depend on the solutions $P_m(r)$ and $Q_m(r)$. Finally, for the radial heat flux using Eqs. (19) and (26–32), one finds

$$q(r) = \gamma S_4(r) + \tau J_4(r) + \int_{r_0}^{\infty} K_{41}(r, r')\, a_1(r)\, \mathrm{d}r' + \int_{r_0}^{\infty} K_{42}(r, r')\, a_2(r')\, \mathrm{d}r' \tag{36}$$

where the simplified expressions for the source terms $S_4(r)$ and $J_4(r)$ and the kernels $K_{41}(r, r')$ and $K_{42}(r, r')$ are given in Appendix A. Following the literature, we will report results for the ratio

$$\frac{q(R, \alpha_t)}{q_{fm}(R, \alpha_t = 1)} = \pi^{1/2}\frac{r^2}{R^2}q(r) \tag{37}$$

where $q(R, \alpha_t) = q(r)$ is the expression of Eq. (36).

Numerical Results and Discussion

We have calculated the radial heat flux ratio, and density and temperature profiles with R (the inverse Knudsen number) ranging from 0.001 to 10 and thermal accommodation coefficients of 0.01, 0.1, 0.2, 0.5, and 1. Table 1 presents the ratio of the total heat flux for different values of thermal accommodation coefficient α_t to the corresponding free molecular heat flux with $\alpha = 1$ [see Eq. (37)]. These results have been compared with the exact expression for this ratio:[11]

$$\frac{q(R, \alpha_t)}{q_{fm}(R, \alpha_t = 1)} = \frac{q(R,1)}{1 + [(1-\alpha_t)/\alpha_t]\,q(R,1)} \tag{38}$$

Table 1 Heat transfer ratio $q(R; \alpha_t)/q_{fm}(R, 1)$

R/α_t	1.0	0.75	0.5	0.2	0.1	0.01
0.001	1.00000	0.75000	0.50000	0.20000	0.10000	0.01000
0.010	0.99600	0.74700	0.49850	0.19960	0.09980	0.00999
0.100	0.97300	0.73425	0.49300	0.19880	0.09970	0.00998
0.500	0.86800	0.67350	0.46450	0.19400	0.09850	0.00998
1.000	0.75900	0.60600	0.43150	0.18800	0.09690	0.00997
2.000	0.59900	0.49950	0.37450	0.17640	0.09370	0.00993
3.000	0.49300	0.42375	0.33050	0.16600	0.09070	0.00990
4.000	0.41900	0.36750	0.29500	0.15660	0.08780	0.00986
5.000	0.36800	0.32775	0.26900	0.14880	0.08530	0.00983
6.000	0.32100	0.29025	0.24300	0.14060	0.08250	0.00979
7.000	0.28700	0.26175	0.22300	0.13360	0.08010	0.00975
8.000	0.25900	0.23850	0.20600	0.12740	0.07780	0.00972
9.000	0.23700	0.21900	0.19150	0.12160	0.07560	0.00968
10.000	0.21700	0.20250	0.17850	0.11620	0.07350	0.00965

and were found to conform precisely to this relation. Tables 2–4 list, respectively, for $\kappa = 1$, the profiles for density and temperature for inverse Knudsen numbers $R = 0.01$, 1.0, and 10, with thermal accommodation coefficient $\alpha_t = 0.01$, 0.5, and 1. These profiles indicate an increase in both density and temperature perturbations as α_t increases.

We have carried out some limited numerical experimentation to access the accuracy of the reported results. Comparison of results obtained by 21-, 41-, and 81-point quadratures showed that, for small $R(\leq 1.0)$, the convergence was quite good whereas, for larger $R(\geq 2.0)$, the accuracy is of the order of 5–10%. The results can be improved either by use of higher-order quadratures or perturbation on the continuum solution.[12]

Our results for the density and temperature distribution in the Knudsen layers are new. Here, no comparable theoretical or experimental results are available and, hence, no verification is possible. For the heat-transfer ratio, however, the results reported here confirm the essential accuracy of the variational results.[4]

The problem of heat transfer from a particle is pertinent to many aspects of aerosol mechanics (heat transfer, growth, thermophoresis), and the work reported here should be useful in further investigations of the Knudsen layers and single particle mechanics.

Appendix A

The final expressions for the source terms $S_m(r)$ and $J_m(r)$ and kernel $K_{mj}(r, r')$ are given by

$$S_1(r) = \frac{1}{\pi^{\frac{1}{2}} r}\int T_2\left[\frac{r^2 - R^2}{t^2} - 1\right]\mathrm{d}t$$

$$S_2(r) = \frac{1}{\pi^{\frac{1}{2}} r}\int \left[T_4(t) - \frac{3}{2}T_2(t)\right]\left[\frac{r^2 - R^2}{t^2} - 1\right]\mathrm{d}t$$

$$S_4(r) = \frac{1}{2\pi^{\frac{1}{2}} r^2}\int \left[T_5(t) - \frac{5}{2}T_3(t)\right]\left[\frac{(r^2 - R^2)^2}{t^3} - t\right]\mathrm{d}t$$

Table 2 Density and temperature profiles for the case of $R = 0.01$

r	$\alpha_t = 0.01$ $a_1(r)$	$\alpha_t = 0.01$ $a_2(r)$	$\alpha_t = 0.50$ $a_1(r)$	$\alpha_t = 0.50$ $a_2(r)$	$\alpha_t = 1.0$ $a_1(r)$	$\alpha_t = 1.0$ $a_2(r)$
0.012	−0.131D−02	0.389D−02	−0.655D−01	0.194D+00	−0.131D+00	0.388D+00
0.056	−0.469D−04	0.130D−03	−0.234D−02	0.647D−02	−0.468D−02	0.129D−01
0.168	−0.682D−05	0.167D−04	−0.340D−03	0.832D−03	−0.680D−03	0.166D−02
0.346	−0.220D−05	0.474D−05	−0.110D−03	0.237D−03	−0.219D−03	0.473D−03
0.586	−0.103D−05	0.200D−05	−0.514D−04	0.100D−03	−0.103D−03	0.200D−03
0.886	−0.587D−06	0.106D−05	−0.293D−04	0.592D−04	−0.585D−04	0.106D−03
1.240	−0.376D−06	0.643D−06	−0.188D−04	0.321D−04	−0.375D−04	0.641D−04
1.650	−0.259D−06	0.427D−06	−0.129D−04	0.213D−04	−0.258D−04	0.426D−04
2.100	−0.187D−06	0.302D−06	−0.936D−05	0.151D−04	−0.187D−04	0.301D−04
2.590	−0.140D−06	0.223D−06	−0.701D−05	0.111D−04	−0.140D−04	0.222D−04
3.120	−0.108D−06	0.169D−06	−0.537D−05	0.946D−05	−0.107D−04	0.169D−04
3.670	−0.840D−07	0.132D−06	−0.419D−05	0.658D−05	−0.838D−05	0.131D−04
4.230	−0.664D−07	0.104D−06	−0.331D−05	0.520D−05	−0.662D−05	0.104D−04
4.810	−0.529D−07	0.833D−07	−0.264D−05	0.416D−05	−0.528D−05	0.831D−05
5.390	−0.424D−07	0.671D−07	−0.212D−05	0.335D−05	−0.423D−05	0.670D−05
5.970	−0.340D−07	0.544D−07	−0.170D−05	0.272D−05	−0.339D−05	0.543D−05
6.530	−0.273D−07	0.442D−07	−0.136D−05	0.221D−05	−0.273D−05	0.441D−05
7.070	−0.219D−07	0.360D−07	−0.109D−05	0.180D−05	−0.218D−05	0.359D−05
7.590	−0.175D−07	0.292D−07	−0.872D−06	0.146D−05	−0.174D−05	0.292D−05
8.060	−0.138D−07	0.237D−07	−0.691D−06	0.118D−05	−0.138D−05	0.237D−05
8.500	−0.109D−07	0.192D−07	−0.542D−06	0.959D−06	−0.108D−05	0.192D−05
8.890	−0.842D−08	0.155D−07	−0.420D−06	0.773D−06	−0.839D−06	0.154D−05
9.230	−0.643D−08	0.124D−07	−0.321D−06	0.621D−06	−0.642D−06	0.124D−05
9.510	−0.483D−08	0.997D−08	−0.241D−06	0.498D−06	−0.482D−06	0.994D−06
9.730	−0.358D−08	0.801D−08	−0.179D−06	0.400D−06	−0.357D−06	0.799D−06
9.890	−0.262D−08	0.650D−08	−0.131D−06	0.325D−06	−0.261D−06	0.649D−06
9.980	−0.197D−08	0.547D−08	−0.983D−07	0.273D−06	−0.196D−06	0.545D−06

$$J_1(r) = \frac{1}{\pi^{\frac{1}{2}} r} \int [T_4(t) - 2T_2(t)] \left[\frac{r^2 - R^2}{t^2} - 1 \right] \mathrm{d}t$$

$$J_2(r) = \frac{1}{\pi^{\frac{1}{2}} r} \int \left[T_6(t) - \frac{7}{2} T_4(t) + 3T_2(t) \right] \left[\frac{r^2 - R^2}{t^2} - 1 \right] \mathrm{d}t$$

$$J_4(r) = \frac{1}{2\pi^{\frac{1}{2}} r^2} \int \left[T_7(t) - \frac{9}{2} T_5(t) + 5T_3 \right] \left[\frac{(r^2 - R^2)^2}{t^3} - t \right] \mathrm{d}t$$

where the integration extend from $(r - R)$ to $(r^2 - R^2)^{\frac{1}{2}}$.

Further,

$$K_{11}(r, r') = 2 \frac{r'}{\pi^{\frac{1}{2}} r} \int \frac{T_1}{t} \mathrm{d}t$$

$$K_{12}(r, r') = \frac{4}{3} \frac{r'}{\pi^{\frac{1}{2}} r} \int \left(T_3 - \frac{3}{2} T_1 \right) \frac{\mathrm{d}t}{t}$$

$$K_{21}(r, r') = 2 \frac{r'}{\pi^{\frac{1}{2}} r} \int \left(T_3 - \frac{3}{2} T_1 \right) \frac{\mathrm{d}t}{t}$$

Table 3 Density and temperature profiles for the case of $R = 1.0$

	$\alpha_t = 0.01$		$\alpha_t = 0.50$		$\alpha_t = 1.0$	
r	$a_1(r)$	$a_2(r)$	$a_1(r)$	$a_2(r)$	$a_1(r)$	$a_2(r)$
1.000	−0.620D − 02	0.122D − 01	−0.268D + 00	0.529D + 00	−0.472D + 00	0.930D + 00
1.090	−0.512D − 02	0.945D − 02	−0.222D + 00	0.409D + 00	−0.390D + 00	0.720D + 00
1.300	−0.389D − 02	0.677D − 02	−0.168D + 00	0.293D + 00	−0.296D + 00	0.515D + 00
1.640	−0.286D − 02	0.474D − 02	−0.124D + 00	0.205D + 00	−0.218D + 00	0.361D + 00
2.100	−0.209D − 02	0.336D − 02	−0.907D − 01	0.145D + 00	−0.160D + 00	0.256D + 00
2.670	−0.155D − 02	0.243D − 02	−0.673D − 01	0.105D + 00	−0.118D + 00	0.185D + 00
3.340	−0.117D − 02	0.181D − 02	−0.507D − 01	0.782D − 01	−0.893D − 01	0.138D + 00
4.120	−0.899D − 03	0.137D − 02	−0.389D − 01	0.595D − 01	−0.685D − 01	0.105D + 00
4.980	−0.700D − 03	0.107D − 02	−0.303D − 01	0.461D − 01	−0.533D − 01	0.811D − 01
5.910	−0.551D − 03	0.838D − 03	−0.239D − 01	0.363D − 01	−0.420D − 01	0.639D − 01
6.910	−0.438D − 03	0.667D − 03	−0.190D − 01	0.289D − 01	−0.334D − 01	0.508D − 01
7.950	−0.351D − 03	0.536D − 03	−0.152D − 01	0.232D − 01	−0.268D − 01	0.408D − 01
9.030	−0.283D − 03	0.432D − 03	−0.122D − 01	0.187D − 01	−0.215D − 01	0.329D − 01
10.100	−0.228D − 03	0.350D − 03	−0.987D − 02	0.152D − 01	−0.174D − 01	0.267D − 01
11.200	−0.184D − 03	0.248D − 03	−0.797D − 02	0.123D − 01	−0.140D − 01	0.217D − 01
12.300	−0.148D − 03	0.231D − 03	−0.642D − 02	0.999D − 02	−0.113D − 01	0.176D − 01
13.400	−0.119D − 03	0.187D − 03	−0.515D − 02	0.809D − 02	−0.906D − 02	0.142D − 01
14.400	−0.949D − 04	0.151D − 03	−0.411D − 02	0.652D − 02	−0.723D − 02	0.115D − 01
15.400	−0.750D − 04	0.121D − 03	−0.325D − 02	0.523D − 02	−0.571D − 02	0.920D − 02
16.300	−0.586D − 04	0.962D − 04	−0.254D − 02	0.416D − 02	−0.446D − 02	0.733D − 02
17.200	−0.451D − 04	0.759D − 04	−0.195D − 02	0.328D − 02	−0.343D − 02	0.578D − 02
17.900	−0.340D − 04	0.592D − 04	−0.147D − 02	0.256D − 02	−0.259D − 02	0.451D − 02
18.500	−0.251D − 04	0.456D − 04	−0.109D − 02	0.197D − 02	−0.191D − 02	0.347D − 02
19.100	−0.179D − 04	0.347D − 04	−0.776D − 03	0.150D − 02	−0.137D − 02	0.264D − 02
19.500	−0.124D − 04	0.261D − 04	−0.535D − 03	0.113D − 02	−0.942D − 03	0.199D − 02
19.800	−0.822D − 05	0.196D − 04	−0.356D − 03	0.848D − 03	−0.626D − 03	0.149D − 02
20.000	−0.543D − 05	0.151D − 04	−0.235D − 03	0.655D − 03	−0.414D − 03	0.115D − 02

Table 4 Density and temperature profiles for the case of $R = 10$

	$\alpha_t = 0.01$		$\alpha_t = 0.50$		$\alpha_t = 1.0$	
r	$a_1(r)$	$a_2(r)$	$a_1(r)$	$a_2(r)$	$a_1(r)$	$a_2(r)$
10.000	−0.400D − 01	0.603D − 01	−0.740D + 00	0.112D + 01	−0.901D + 00	0.136D + 01
10.300	−0.382D − 01	0.572D − 01	−0.707D + 00	0.106D + 01	−0.861D + 00	0.129D + 01
11.000	−0.351D − 01	0.522D − 01	−0.650D + 00	0.966D + 00	−0.791D + 00	0.118D + 01
12.000	−0.313D − 01	0.463D − 01	−0.580D + 00	0.858D + 00	−0.706D + 00	0.104D + 01
13.500	−0.273D − 01	0.402D − 01	−0.505D + 00	0.745D + 00	−0.615D + 00	0.907D + 00
15.300	−0.234D − 01	0.344D − 01	−0.433D + 00	0.636D + 00	−0.527D + 00	0.774D + 00
17.400	−0.198D − 01	0.290D − 01	−0.366D + 00	0.537D + 00	−0.446D + 00	0.654D + 00
19.800	−0.166D − 01	0.243D − 01	−0.307D + 00	0.450D + 00	−0.374D + 00	0.548D + 00
22.600	−0.139D − 01	0.203D − 01	−0.256D + 00	0.375D + 00	−0.312D + 00	0.457D + 00
25.500	−0.115D − 01	0.169D − 01	−0.213D + 00	0.312D + 00	−0.259D + 00	0.380D + 00
28.700	−0.953D − 02	0.140D − 01	−0.176D + 00	0.259D + 00	−0.215D + 00	0.315D + 00
32.000	−0.787D − 02	0.115D − 01	−0.146D + 00	0.214D + 00	−0.177D + 00	0.260D + 00
35.400	−0.646D − 02	0.952D − 02	−0.120D + 00	0.176D + 00	−0.146D + 00	0.214D + 00
38.800	−0.529D − 02	0.781D − 02	−0.978D − 01	0.145D + 00	−0.119D + 00	0.176D + 00
42.300	−0.430D − 02	0.638D − 02	−0.796D − 01	0.118D + 00	−0.969D − 01	0.144D + 00
45.800	−0.347D − 02	0.518D − 02	−0.642D − 01	0.959D − 01	−0.782D − 01	0.117D + 00
49.200	−0.278D − 02	0.418D − 02	−0.514D − 01	0.773D − 01	−0.626D − 01	0.941D − 01
52.400	−0.220D − 02	0.333D − 02	−0.407D − 01	0.616D − 01	−0.495D − 01	0.750D − 01
55.500	−0.172D − 02	0.262D − 02	−0.317D − 01	0.485D − 01	−0.386D − 01	0.591D − 01
58.400	−0.131D − 02	0.203D − 02	−0.243D − 01	0.376D − 01	−0.296D − 01	0.458D − 01
61.000	−0.985D − 03	0.155D − 02	−0.182D − 01	0.286D − 01	−0.222D − 01	0.348D − 01
63.300	−0.716D − 03	0.115D − 02	−0.133D − 01	0.212D − 01	−0.161D − 01	0.258D − 01
65.400	−0.501D − 03	0.825D − 03	−0.926D − 02	0.153D − 01	−0.113D − 01	0.186D − 01
67.100	−0.332D − 03	0.571D − 03	−0.614D − 02	0.106D − 01	−0.748D − 02	0.129D − 01
68.400	−0.205D − 03	0.378D − 03	0.379D − 02	0.699D − 02	−0.461D − 02	0.850D − 02
69.300	−0.114D − 03	0.238D − 03	−0.211D − 02	0.440D − 02	−0.257D − 02	0.535D − 02
69.900	−0.561D − 04	0.146D − 03	−0.104D − 02	0.270D − 02	−0.126D − 02	0.329D − 02

$$K_{22}(r,r') = \frac{4}{3}\frac{r'}{\pi^{\frac{1}{2}} r}\int\left(T_5 - 3T_3 + \frac{9}{4}T_1\right)\frac{\mathrm{d}t}{t}$$

$$K_{41}(r,r') = \frac{r'}{\pi^{\frac{1}{2}} r^2}\int\left(T_4 - \frac{5}{2}T_2\right)\left(\frac{r^2 - r'^2}{t^2} + 1\right)\mathrm{d}t$$

$$K_{42}(r,r') = \frac{2}{3}\frac{r'}{\pi^{\frac{1}{2}} r^2}$$

$$\int\left(T_6 - 4T_4 + \frac{15}{4}T_2\right)\left[\frac{r^2 - r'^2}{t^2} + 1\right]\mathrm{d}t$$

The limit of integration in the preceding integrals are from $|r - r'|$ to $(r^2 - R^2)^{\frac{1}{2}} + (r'^2 - R^2)^{\frac{1}{2}}$, and the argument of T_n functions is $t = |r - r'|$.

The T_n are Abramowitz functions defined by

$$T_n(x) = \int_0^\infty t^n \exp\left(-t^2 - \frac{x}{t}\right)\mathrm{d}t$$

Appendix B

The elements of matrix $\underline{\underline{D}}$ are given by

$$D_{11} = 1 + \frac{2}{R^2}\sum_{m=1}^{2}\int_R^\infty rP_m(r)\chi_{m,1}(r)\,\mathrm{d}r$$

$$D_{12} = \frac{2}{R^2}\sum_{m=1}^{2}\int_R^\infty rQ_m(r)\chi_{m,1}(r)\,\mathrm{d}r$$

$$D_{21} = \frac{(1-\alpha)}{R^2}\sum_{m=1}^{2}\int_R^\infty rP_m(r)\chi_{m,2}(r)\,\mathrm{d}r$$

$$D_{22} = 1 + \frac{(1-\alpha)}{R^2}\sum_{m=1}^{2}\int_R^\infty rQ_m(r)\chi_{m,2}(r)\,\mathrm{d}r$$

where

$$\chi_{m,1}(r) = \{[\psi_m(c;r), c^2]\}$$

$$x_{m,2}(r) = \{[\psi_m(c;r), c^2(c^2 - 2)]\}$$

and the inner product is defined by

$$((\psi_m, f)) = \int_{r-R}^{(r^2-R^2)^{\frac{1}{2}}}\left(1 - \frac{r^2 - R^2}{t^2}\right)\mathrm{d}t$$

$$\int_0^\infty \psi_m(c;r)\exp\left(-c^2 - \frac{t}{c}\right) f\,\mathrm{d}c$$

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